GCSE Maths – Algebra

Sketching Graphs – Linear, Quadratic, Cubic and Reciprocal

Notes

WORKSHEET

Sketching Graphs - Linear, Quadratic, Cubic and Reciprocal

Linear graphs

A linear graph is a **straight line**, with a line equation of the form

$$y = mx + c,$$

where m is the **gradient** and c is the **y-intercept**.

To **sketch** a linear graph, begin by completing a **table of values.** Using the equation of the line, the table shows the **corresponding** y value for chosen x values.

Example: Construct the table of values for the general formula $y = mx + c$ between $x = -2$ and $x = 2$

Substitute each x value into the formula to obtain the relevant y value:

x	-2	-1	0	1	2
$y = mx + c$	$-2m + c$	$-m + c$	$0 + c = c$	$m + c$	$2m + c$

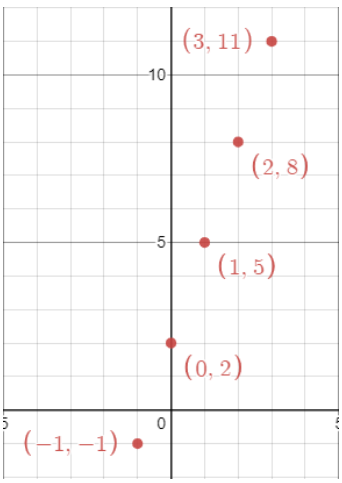
Points from a table of values can then be plotted on a graph and connected with a straight line.

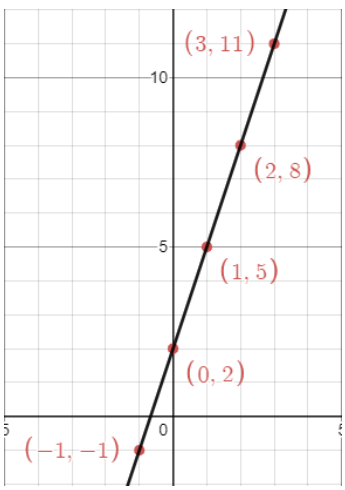
Example: Sketch a graph of the function $y = 3x + 2$ between $x = -1$ and $x = 3$

First, create the table of values for the required values of x:

x	-1	0	1	2	3
$y = 3x + 2$	-1	2	5	8	11

Plot these values on a graph by plotting the coordinates $(-1, -1)$, $(0, 2)$, $(1, 5)$, $(2, 8)$ *and* $(3, 11)$. *Then draw a line connecting these points. We can check that the gradient of the graph is indeed* 3 *and the y-intercept is* 2.





Quadratic graphs

A quadratic graph is shaped like a **parabola**: ∪ or ∩.
Its equation is of the form

$$y = ax^2 + bx + c,$$

where $a \neq 0$.

- If $a < 0$ then the graph will have the shape ∩.
- If $a > 0$ then the graph will have the shape ∪.

Sketching a quadratic graph is a similar process to sketching a linear graph. If using a table of values, you proceed by creating the **table of values**, plotting the **points**, and then connecting them with a **smooth curve**.

Example: Plot $y = x^2 - 2x + 1$ between $x = -2$ and $x = 2$

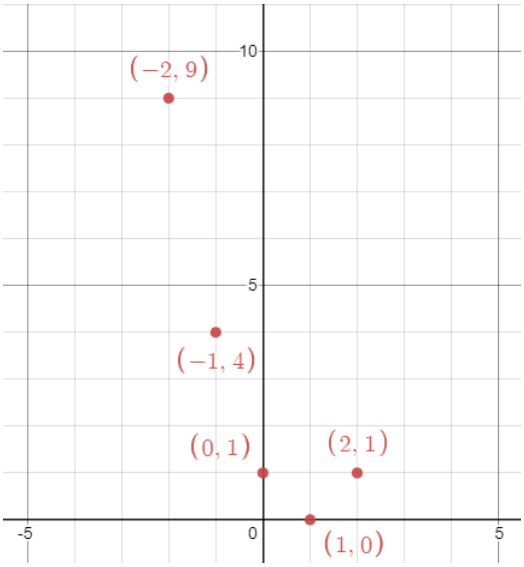
First, create the table of values for the values of x:

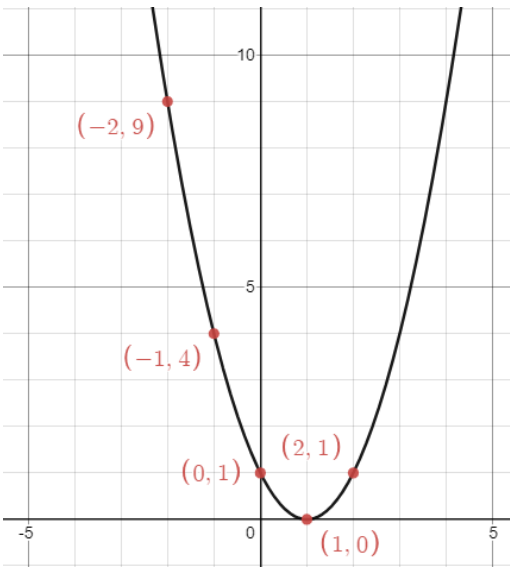
x	-2	-1	0	1	2
$y = x^2 - 2x + 1$	9	4	1	0	1

Plot these values on a graph by plotting the coordinates

$$(-2, 9),\ (-1, 4),\ (0, 1),\ (1, 0) \text{ and } (2, 1).$$

Then draw a curve connecting these points:



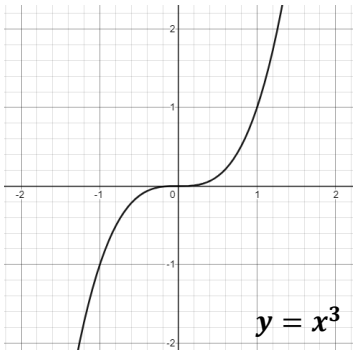


Cubic graphs

Cubic graphs can have more than one **turning point** and have equations of the form

$$y = ax^3 + bx^2 + cx + d,$$

where $a \neq 0$.



Cubic graphs have a **shape** similar to the curve on the right.

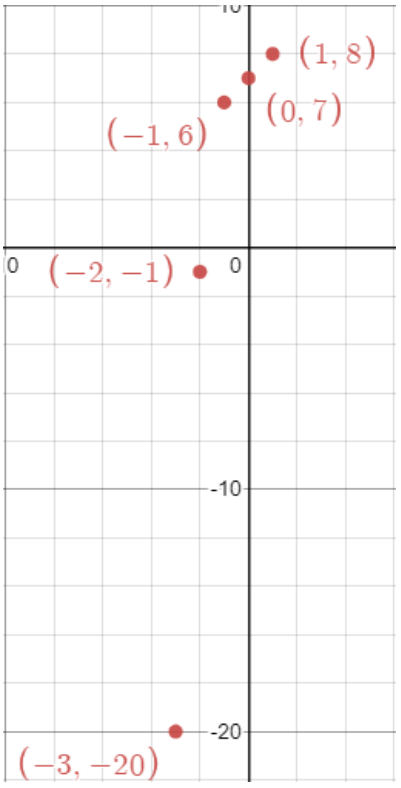
Sketching cubics can be done similarly to sketching a quadratic curve. Simply complete a **table of values**, plot the **points** and connect them with a **smooth curve**.

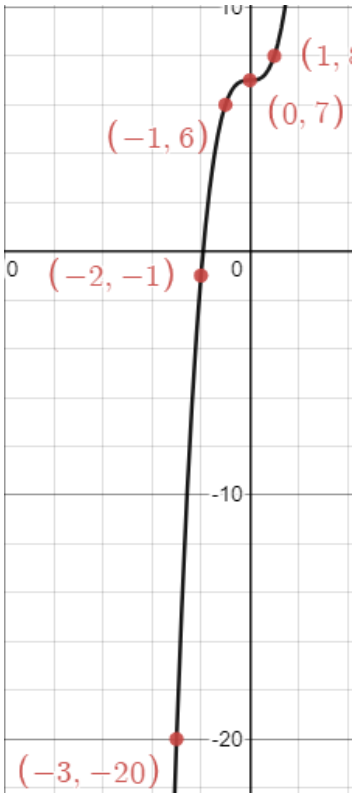
Example: Plot $y = x^3 + 7$ between $x = -3$ and $x = 1$

First, complete a table of values for the required x values:

x	-3	-2	-1	0	1
$y = x^3 + 7$	-20	-1	6	7	8

Plot these values on a graph by plotting the coordinates $(-3, 20)$, $(-2, -1)$, $(-1, 6)$, $(0, 7)$ *and* $(1, 8)$. *Then draw a curve connecting these points:*



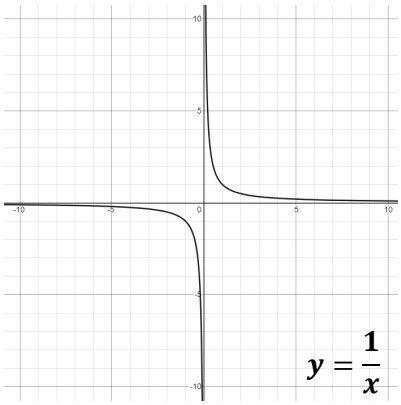


Reciprocal graphs

Reciprocal graphs are of the form

$$y = \frac{1}{x}$$

where $x \neq 0$.



The value of x cannot be 0 otherwise we would get $\frac{1}{0}$, which is **undefined**.

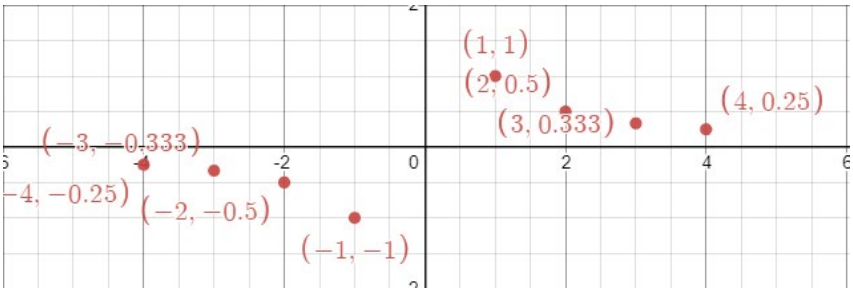
When **sketching** reciprocal graphs of this form you need to keep in mind there will be a **discontinuity at $x = 0$**. Apart from that, you proceed in the same way as the previous graphs: construct a **table of values**, plot the **points** and then connect them with a **smooth curve**.

Example: Plot $y = \frac{1}{x}$ between $x = -4$ and $x = 4$.

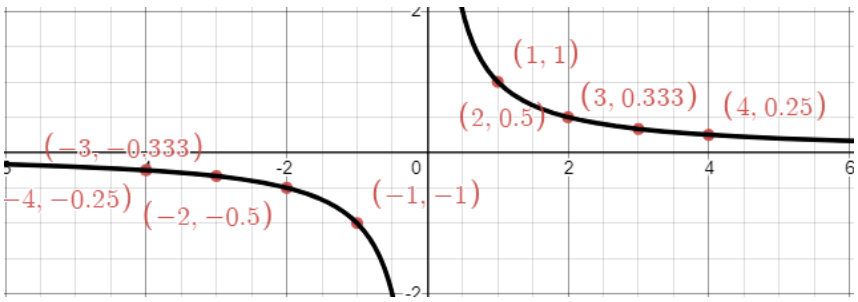
First, complete a table of values for the required x values:

x	-4	-3	-2	-1	0	1	2	3	4
$y = \frac{1}{x}$	-0.25	-0.33	-0.5	-1	Undefined	1	0.5	0.33	0.25

Plot these values on a graph by plotting the coordinates $(-4, -0.25)$, $(-3, -0.33)$, $(-2, -0.5)$, $(-1, -1)$, $(1, 1)$, $(2, 0.5)$, $(3, 0.33)$ *and* $(4, 0.25)$.



Draw a smooth curve connecting these points:



Linear, Quadratic, Cubic and Reciprocal Graphs – Practice Questions

1) Sketch $y = 2x^2 + x + 2$ between $x = -3$ and $x = 2$.

2) Sketch $y = x^3 + 2x + 1$ between $x = -3$ and $x = 3$.

3) Sketch $y = \frac{1}{x} + 2$ between $x = -5$ and $x = 5$.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.